GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

June 1988

Alberta
EDUCATION

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Three written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example

This examination is for the subject area of

A. Chemistry
B. Biology
C. Physics
D. Mathematics

Answer Sheet

A	B	C	D
①	②	③	●

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work. NO MARKS will be given for work done on the tear-out sheets.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JUNE 1988

PART A

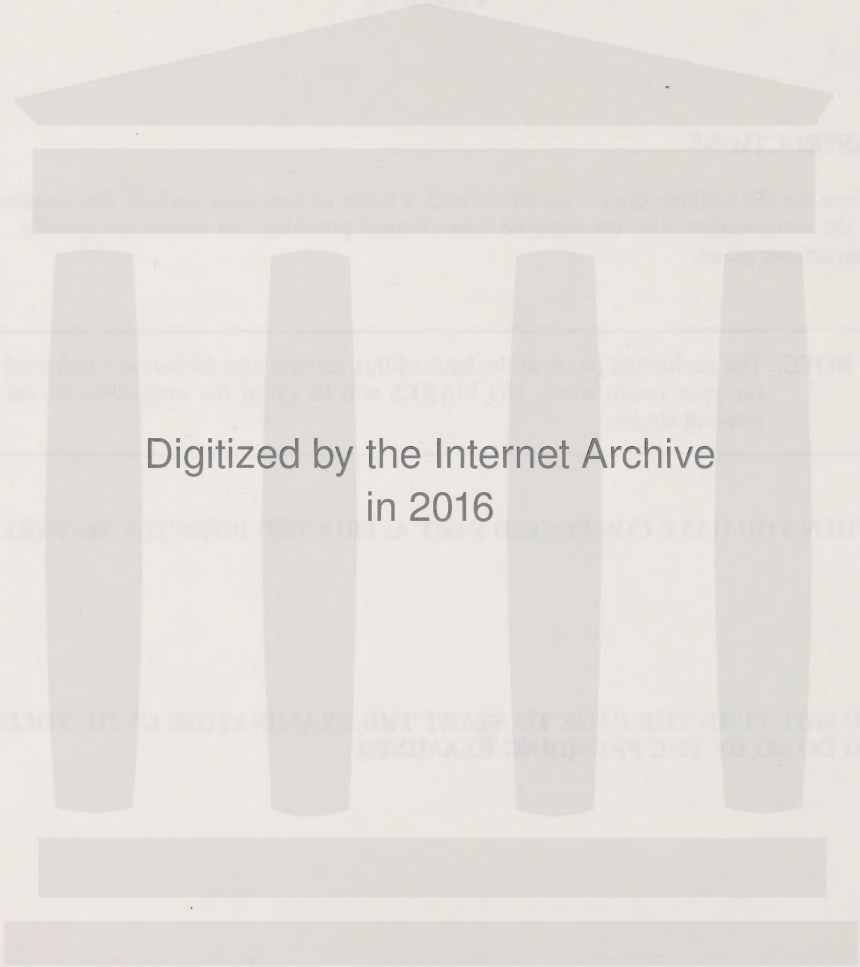
INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work. NO MARKS will be given for work done on the tear-out sheets.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER



1. In ΔABC, if $a = 20$ cm, $b = 25$ cm, and $c = 30$ cm, then the measure of $\angle A$ to the nearest degree is

A. 86°

B. 68°

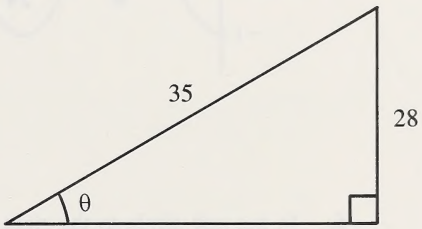
C. 53°

D. 41°

2. Three camps are set up, one at each vertex of a triangle. Camp A is 9.00 km from Camp B, and Camp B is 13.00 km from Camp C. If $\angle BAC$ is 50°, then the distance between camps A and C would be

A. 16.81 km
B. 15.81 km
C. 14.75 km
D. 9.98 km

3. In the triangle shown at the right, $\sec \theta$ is equal to



A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{5}{4}$

D. $\frac{5}{3}$

4. The exact value of $6\cos\left(\frac{11\pi}{6}\right) - 2\sin\left(\frac{5\pi}{4}\right)$ is

A. $-3 - \sqrt{2}$

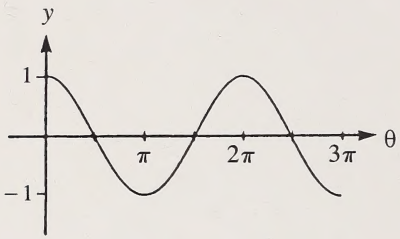
B. $-3 + \sqrt{2}$

C. $3\sqrt{3} - \sqrt{2}$

D. $3\sqrt{3} + \sqrt{2}$

5. The graph of the function $y = \cos\theta$ is

A.



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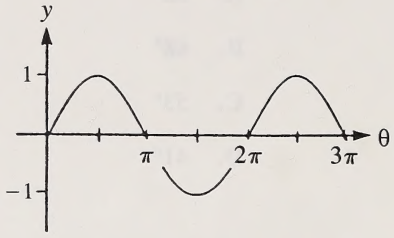

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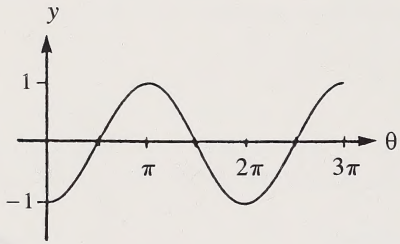
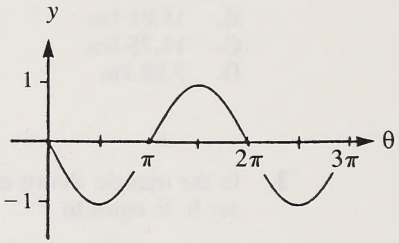
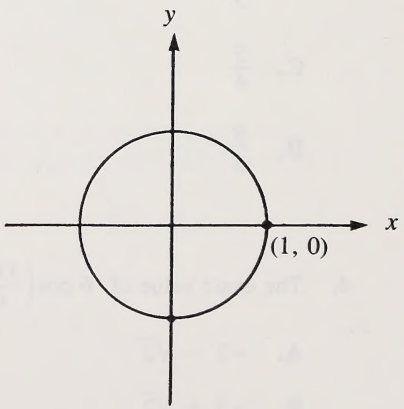


D.



6. In the unit circle to the right, a path with length $-\frac{14\pi}{6}$ has an initial point at (1, 0) and a terminal point at



A. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

B. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

C. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

D. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

7. If $\cos(\pi + \theta) = \frac{3}{11}$, then $\cos\theta$ is equal to

A. $\frac{14}{11}$

B. $\frac{3}{11}$

C. $-\frac{3}{11}$

D. $-\frac{8}{11}$

8. In radian measure, two-thirds of a degree is equivalent to

A. $\frac{\pi}{9}$

B. $\frac{2\pi}{9}$

C. $\frac{\pi}{270}$

D. $\frac{\pi}{450}$

9. In the identity $\tan B \sin B + \cos B = \frac{1}{x}$, the value of x must be

A. $\cos B$
B. $\sin B$
C. $\tan B$
D. $\cot B$

10. If $\cos^2\theta - 3\sin^2\theta = 0$, one value of θ is

A. 330°

B. 240°

C. 120°

D. 60°

11. If $\cot\theta = -\frac{3}{4}$ and $\sin\theta$ is negative, then the value of the expression $(\sin\theta - \cos\theta)^2$ is

A. $-\frac{7}{5}$

B. $\frac{6}{25}$

C. $\frac{7}{5}$

D. $\frac{49}{25}$

12. The centre of the circle defined by $x^2 + y^2 + 8x - 14y + 56 = 0$ is at

A. $(-8, 14)$
B. $(-4, 7)$
C. $(4, -7)$
D. $(8, -14)$

13. If an ellipse is centred at the origin with one focus at (5, 0) and the sum of the focal radii is 20, then the equation of the ellipse is

A. $\frac{x^2}{100} + \frac{y^2}{75} = 1$

B. $\frac{x^2}{10} + \frac{y^2}{5} = 1$

C. $\frac{x^2}{5} + \frac{y^2}{20} = 1$

D. $\frac{x^2}{4} + \frac{y^2}{20} = 1$

14. The statement that is true for all hyperbolas is

A. $c < a$
B. $b < a$
C. $a < b$
D. $a < c$

15. The slope of the line represented by $2x + 3y = 6$ is

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $-\frac{3}{2}$

D. $-\frac{2}{3}$

16. The equation of a circle with centre at $(1, -2)$ and tangent to the line $y = 6$ is

A. $x^2 + y^2 - x + 2y - 69 = 0$

B. $x^2 + y^2 + x - 2y - 69 = 0$

C. $x^2 + y^2 - 2x + 4y - 59 = 0$

D. $x^2 + y^2 + 2x - 4y - 59 = 0$

17. The equation of the parabola of the form $y^2 = 4px$ with its focus on the line $5x + 8y - 15 = 0$ is

A. $y^2 = -20x$

B. $y^2 = 3x$

C. $y^2 = 12x$

D. $y^2 = 20x$

18. A hyperbola has an equation of $\frac{y^2}{64} - \frac{x^2}{9} = 1$. The foci are at

A. $(0, \pm\sqrt{55})$

B. $(0, \pm\sqrt{73})$

C. $(\pm\sqrt{55}, 0)$

D. $(\pm\sqrt{73}, 0)$

19. The vertex of the parabola $(y - 2)^2 = 4x - 12$ is at

A. (2, 3)
B. (3, 2)
C. (3, 3)
D. (4, 2)

20. The equation of a parabola with a focus at $(-3, 0)$ and a directrix of $x = 3$ is

A. $x^2 = -12y$

B. $y^2 = -12x$

C. $x^2 = 12y$

D. $y^2 = 12x$

21. For the ellipse $\frac{x^2}{25} + \frac{y^2}{169} = 1$, the end-points of the major and minor axes respectively are at

A. $(0, \pm 13), (\pm 5, 0)$

B. $(0, \pm 26), (\pm 10, 0)$

C. $(\pm 13, 0), (\pm 5, 0)$

D. $(\pm 26, 0), (0, \pm 10)$

22. A hyperbola with its vertices on the x-axis and one asymptote defined by the equation $6y = 5x$ is

A. $\frac{x^2}{25} - \frac{y^2}{36} = 1$

B. $\frac{y^2}{25} - \frac{x^2}{36} = 1$

C. $\frac{y^2}{36} - \frac{x^2}{25} = 1$

D. $\frac{x^2}{36} - \frac{y^2}{25} = 1$

23. An ellipse has its centre at the origin and foci on the y-axis. If $a = 5$ and $b = 2$, then one focus is at

A. $(0, \sqrt{21})$

B. $(\sqrt{21}, 0)$

C. $(0, \sqrt{29})$

D. $(\sqrt{29}, 0)$

24. The major axis of an ellipse is five times the length of the minor axis, and $P(2, 5)$ is a point on the ellipse. The equation of the ellipse with centre $(0, 0)$ and foci on the y-axis is

A. $x^2 + 25y^2 = 629$

B. $x^2 + 25y^2 = 125$

C. $x^2 + 5y^2 = 129$

D. $25x^2 + y^2 = 125$

25. The set of all points P, such that the distance from P to a fixed line is the same as the distance from P to a fixed point not on the line, is

A. a circle
B. an ellipse
C. a parabola
D. a hyperbola

26. The sequence that could be arithmetic is

A. $-2, -4, -8, \ldots$

B. $2, 3, 4, \ldots$

C. $\sqrt{2}, 2, 2\sqrt{2}, 4, \ldots$

D. $1, 0, -1, 0, -2, \ldots$

27. The number of terms in $\sum_{n=3}^{15} n^2$ is

A. 5
B. 12
C. 13
D. 15

28. The sum of money that will amount to \$3500 in 20 years at 4% per annum compounded annually, to the nearest dollar, is

A. \$1597
B. \$1600
C. \$1603
D. \$1606

29. A university student plans to make a deposit at the end of each year for four years into an account that pays 7% per annum compounded annually. If he wishes to have \$5500 in the account immediately following the fourth deposit, then the amount that should be invested each year is

A. \$1201.25
B. \$1238.75
C. \$1278.75
D. \$1471.25

30. In a geometric series of four terms, the sum of the first two terms is 8 and the sum of the last two terms is 72. If the common ratio is positive, the first term must be

A. 2
B. 3
C. 4
D. 5

31. The first four terms of the sequence defined by $f_n = \frac{(-1)^n}{n+1}$, $n \in N$ are

A. $-\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}$

B. $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}$

C. $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}$

D. $-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}$

32. If the sum of an infinite geometric series is 3 and the common ratio is $\frac{1}{3}$, then the first term is

A. $\frac{9}{2}$

B. 2

C. 1

D. $\frac{1}{2}$

33. For an arithmetic series $t_1 = 2$ and $t_{100} = 40$. The sum of the first 100 terms is

A. 4200
B. 4158
C. 2100
D. 2079

34. A measure of central tendency is the

A. mode
B. range
C. sample size
D. percentile rank

35. The mean time it takes for customers to be served in a cafe is 10 min with a standard deviation of 2 min. Assuming a normal distribution, if the cafe serves 200 customers, the number served within 12 min is

A. 32
B. 68
C. 136
D. 168

36. The mean life of a brand of batteries is given as 60 h with a standard deviation of 4 h. If the distribution is normal, the probability that one of these batteries will last less than 55 h is

A. 0.8136
B. 0.4132
C. 0.3944
D. 0.1056

Use the following information to answer questions 37 to 39.

Every Grade 11 student in Terry's high school wrote four achievement examinations. The results on the examinations were normally distributed. Terry's results are summarized in the following table.

Subject	Mean Mark	Terry's Mark	Standard Deviation
Biology	68	80	9
Physics	53	73	5
Chemistry	70	56	7
Mathematics	61	86	15

37. Considering all the data in the table, Terry ranks highest in

A. Biology
B. Physics
C. Chemistry
D. Mathematics

38. On a scale with a mean of 500 and a standard deviation of 120, Terry's mark in Chemistry would be

A. 260
B. 380
C. 400
D. 486

39. Terry's mean z-score is

A. 1.00
B. 1.19
C. 1.25
D. 1.50

40. For a normal distribution of masses, the mean is 18 g and the standard deviation is 8 g. The probability of obtaining a mass that is greater than 24 g is

A. 0.2266
B. 0.2734
C. 0.7266
D. 0.7734

41. For packaged cookies, the mean mass of each piece is 55 g and the standard deviation is 1.3 g. If a normal distribution is assumed and if the cookies must have a mass between 52.4 g and 57.6 g, the percentage that will be rejected is

A. 95.44
B. 47.72
C. 9.12
D. 4.56

42. In a population with a normal distribution, 60 elements are between $\mu - \sigma$ and $\mu + \sigma$. To the nearest whole number, the expected size of the population is

A. 89
B. 88
C. 44
D. 41

43. Of the 800 people at a camp, 500 are boys and 300 are girls. A representative sample is needed to determine the best time for a corn roast. If 56 people are to be polled, the number that should be boys is

A. 21
B. 28
C. 34
D. 35

44. If $\log_9(27) = y$, then y equals

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 3

D. 9^{27}

45. Expressed as a single logarithm, $3\log_5(4) + \log_5(2) - \log_5(4)$ is

A. $\log_5(10)$

B. $\log_5(20)$

C. $\log_5(32)$

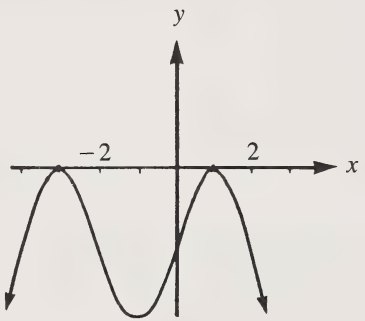
D. $\log_5(62)$

46. In the synthetic division shown to the right, the value of n is

$$\begin{array}{r|cccc} -2 & n & 0 & 1 & 3 \\ & & -4 & 8 & -18 \\ \hline & n & -4 & 9 & -15 \end{array}$$

A. -4
B. -2
C. 2
D. 4

47. The sketch to the right could be the graph of the function



A. $y = (x^2 - 3)(x^2 + 1)$

B. $y = (x^2 + 3)(x^2 - 1)$

C. $y = -(x - 3)^2(x + 1)^2$

D. $y = -(x + 3)^2(x - 1)^2$

48. When $x^3 - ax^2 - bx + 6$ is divided by $x - 3$, the remainder is zero but when it is divided by $x - 2$, the remainder is -4. The values of a and b respectively are

A. 2, 5
B. $-2, -5$
C. 8, -15
D. -8, 15

49. If one x-intercept of the graph of $y = x^3 + 3x^2 - 10x + m$ is -2, then the other x-intercepts are

A. 2 and -12
B. 3 and -4
C. 4 and -3
D. 12 and -2

50. If $x^2 - \frac{2}{3}x + k$ is to be a perfect square, then the value of k must be

A. $\frac{4}{9}$

B. $\frac{1}{3}$

C. $\frac{1}{9}$

D. $-\frac{1}{9}$

51. The value of k that will make $x^3 - 6x^2 - 5x + k$ divisible by $x - 1$ is

A. 10
B. 2
C. -2
D. -10

52. If $x - 1$ and $x + 2$ are factors of $P(x) = x^4 - 2x^2 + 3x - 2$, then another factor of $P(x)$ is

A. $x^2 - x + 2$

B. $x^2 + x + 2$

C. $x^2 - x + 1$

D. $x^2 + x + 1$

YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work. NO MARKS will be given for work done on the tear-out sheets.

TOTAL MARKS: 13

START PART B IMMEDIATELY

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(4 marks)

1. Given the sequence 12, x, 27, . . . , find the value(s) of x that will make

a) the sequence geometric

$x =$ _____

b) the sequence arithmetic

$x =$ _____

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(4 marks)

Use the following information to answer question 2.

Amount of Carbon-14 after t years

$M(t) = M_0\left(2^{-\frac{t}{5760}}\right)$, where $M(t)$ = amount after t years

M_0 = starting amount

t = number of years

2. The remains of a plant were found to have 0.32 g of C^{14} (Carbon-14). It was estimated that the sample originally had 1.28 g of C^{14}. How many years ago was the plant alive?

t = _____

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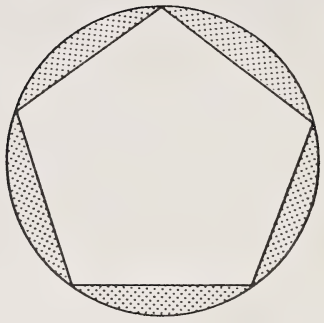
(5 marks)

3. a) A regular pentagon is cut from a circular disc that has a diameter of 14.0 cm. Find the area of the largest possible regular pentagon that can be produced in this way, correct to the nearest tenth of a square centimetre.



Area is _____

b) What percentage of the disc is cut off to produce the largest possible regular pentagon, correct to one decimal place?

Percentage cut off is _____

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION

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M1 M2 M3 M4

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MATHEMATICS 30

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